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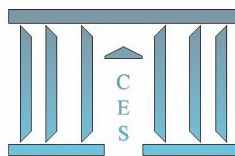
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**Robust Viable Analysis of an Ecosystem Model**

Esther REGNIER, Michel DE LARA

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# Robust Viable Analysis of an Ecosystem Model

Esther REGNIER \*

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## Abstract

The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the adoption of an ecosystem approach. In this perspective, we propose a theoretical management framework that deals jointly with three issues: i) ecosystem dynamics, ii) conflicting issues of production and preservation and iii) robustness with respect to dynamics uncertainties. We consider a discrete-time two-species dynamic model, where states are biomasses, and where two controls act as harvesting efforts of each species. Uncertainties take the form of disturbances affecting each species growth factors, and are assumed to take their values in a known given set. We define the robust viability kernel as the set of initial species biomasses such that at least one harvesting strategy guarantees minimal production and preservation levels for all times, whatever the uncertainties. We apply our approach to the anchovy-hake couple in the Peruvian upwelling ecosystem. We find that accounting for uncertainty sensibly shrinks the deterministic viability kernel (without uncertainties). We comment on the management implications of comparing robust viability kernels (with uncertainties) and the deterministic one (without uncertainties).

**Key words:** viability; uncertainty; robustness; sustainability; fisheries; Peruvian upwelling ecosystem.

## 1 Introduction

There is a growing demand for moving from single species management schemes to an ecosystemic approach of fisheries management [Garcia, Zerbi, Aliaume, Chi, and Lasserre, 2003]. The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the application of an ecosystem approach by 2010. However the dynamics of ecosystems are complex and poorly understood. The ecosystem approach of fisheries faces many issues, ranging from the high cost of the science required (developing data collection, analytical tools,

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and models) to the practical difficulties of changing the governance system and processes [Sainsbury, Punt, and Smith, 2000, Cury, Mullon, Garcia, and Shannon, 2005].

Furthermore, uncertainty inherent to fisheries is recognized to play an important role in the failure of management regimes. Fisheries modeling requires estimations of stock status and total withdrawal from stock; such information remains imprecise and error prone. Uncertainty can also concern the structure and dynamics of ecosystems, which are poorly known. At last, uncertain climatic hazards or technical progress are likely to affect fisheries productivity. Some claim that fishing decreases the resilience of fish populations, rendering them more vulnerable to environmental change [Lauck, Clark, Mangel, and Munro, 1998] and, that not accounting for uncertainty can lead to excessive harvest of a resource [Hilborn and Walters, 1992].

We propose a management framework grounded in viability theory that deals jointly with i) ecosystem dynamics, ii) conflicting issues of production and preservation and iii) robustness with respect to dynamics uncertainties.

We set forward the robust viability theory [De Lara and Doyen, 2008] as a relevant approach to address dynamical control problems under constraints with uncertainty. The theory concentrates on initial states as follows. Starting from a so-called *robust viable state*, there exists a control strategy guaranteeing constraints — here production and preservation objectives — for all dates of a time span, and for all uncertainties. The set of robust viable states is called the *robust viability kernel*. What characterizes the robust viability theory is that no trade-offs are allowed between pursued objectives or time periods: all constraints must be satisfied for all times, whatever the uncertainties. This approach is convenient in the situations where poor information is available on the distribution of uncertainties since it does not require to assign probabilistic assumptions to uncertainty scenarios, as failure or success with respect to scenarios are the only options.

We apply this theory to a discrete-time two-species dynamical model, where states are biomasses and where two harvesting efforts act as controls. Uncertainties take the form of disturbances affecting each species growth factors, and are assumed to take their values in a known given set (we consider different uncertainty sets in order to appraise the sensibility of our results to uncertainties). Constraints are imposed for each species: a minimum safe biomass level, usually identified by biologists, and a minimum required harvesting level assumed to ensure economic needs. These thresholds are generally set constant over time, implying that all generations are subject to the same constraints. This formalization of the problem is in line with the egalitarian vision of resource exploitation advocated by Rawls [1971], Solow [1974]. In fact, Doyen and Martinet [2012] demonstrate that the viability framework allows to characterize the maximin path as a particular viable trajectory. Going further, the authors explain that “whenever the solution of a given optimization problem can be formulated in terms of a viability kernel, the solution inherits the properties of the kernel”. Besides, given that wildlife populations often display wide fluctuations in an unpredictable way, fisheries management goals and schemes should be updated regularly, in accordance to the new data

on stock assessments. Hence, given management exercises with a time frame of a couple of years, keeping sustainability constraints unchanged appears sensible in view of the lifetime of one generation.

Thus, starting from a robust viable biomass couple, it is possible to drive the system on a sustainable path along which catches and biomasses stand above production and biological minimums, despite uncertainties.

Reducing uncertainties to zero amounts to dressing the problem as deterministic [Aubin, 1991]. Comparison of deterministic and robust viable states shades light on the distance between the outcomes of these two extreme approaches: ignoring uncertainty vs. hedge against any risk. We do not advocate the robust viability approach as a fully suitable decision tool for fishery management, since the complete elimination of risk involves economic costs for society, that are not justified when no catastrophic or irreversible events are expected, or when their likeliness is low. Our aim is to emphasize the impact of adopting a precautionary approach with respect to uncertainty on management possibilities of a harvested ecosystem, that arise from a same methodology. It is also an opportunity to emphasize the different analysis and the wide range of information that can be derived from the viability framework to support decision making in the sustainable management of fisheries.

Several studies have applied the deterministic viable control method to the management of natural resources [Martinet and Doyen, 2007] and, in particular, to fisheries management [Béné, Doyen, and Gabay, 2001, Eisenack, Sheffran, and Kropp, 2006, De Lara, Doyen, Guilbaud, and Rochet, 2007, Martinet, Doyen, and Thébaud, 2007, Chapel, Deffuant, Martin, and Mullon, 2008, Martinet, Thébaud, and Rapaport, 2010] as well as the stochastic viable framework [Doyen, De Lara, Ferraris, and Pelletier, 2007, De Lara and Martinet, 2009, Doyen, Thébaud, Béné, Martinet, Gourguet, Bertignac, Fifas, and Blanchard, 2012]. Yet, very few studies have undertaken a robust approach to these issues [Béné and Doyen, 2003].

The paper is organized as follow. Section 2 introduces a generic class of harvested nonlinear ecosystem models, the sustainability constraints, and presents the concept of robust viability kernel. The deterministic viability kernel is also defined for comparison purpose. In Section 3, we proceed with an application of the robust and deterministic viability analysis to the Peruvian hake-anchovy upwelling ecosystem between 1971 and 1981. We numerically compute robust viability kernels, stemming from different uncertainty sets; we compare them to the deterministic viability kernel, whose expression is obtained analytically. Section 4 concludes.

## 2 The Robust Viability Approach

In what follows, we present a class of generic harvested nonlinear ecosystem models with uncertainty. Next, we introduce the concept of robust viable state, that is, a state starting from which conservation and production constraints can be guaranteed over a given time span, despite of uncertainty. Then we define the set of deterministic viable states — states guaranteeing conservation and production constraints in absence

of uncertainties — for which we are able to provide an analytical expression.

## 2.1 A generic ecosystem model with uncertainty and the associated sustainability constraints

We consider a discrete-time dynamic model with two species, each targeted by a specific fleet.<sup>1</sup> Each species is described by its biomass: the two-dimensional state vector  $(y, z)$  represents the biomass of both species. The two-dimensional control vector  $(v_y, v_z)$  comprises the harvesting effort for each species, respectively, each lying in  $[0, 1]$ . Two terms  $\varepsilon_y$  and  $\varepsilon_z$  correspond to uncertainties affecting each species, respectively. The discrete-time control dynamical system we consider is given by

$$\begin{cases} y(t+1) &= y(t)\mathcal{R}_y(y(t), z(t), \varepsilon_y(t))(1 - v_y(t)) , \\ z(t+1) &= z(t)\mathcal{R}_z(y(t), z(t), \varepsilon_z(t))(1 - v_z(t)) , \end{cases} \quad (1)$$

where  $t$  stands for time (typically, periods are years), and ranges from the initial time  $t_0$  to the time horizon  $T$  (where  $T \geq t_0 + 2$ ). The two functions  $\mathcal{R}_y : \mathbb{R}^3 \rightarrow \mathbb{R}$  and  $\mathcal{R}_z : \mathbb{R}^3 \rightarrow \mathbb{R}$  represent biological *growth factors*, and are supposed to be continuous. The property that the growth factor  $\mathcal{R}_y(y, z, \varepsilon_y)$  of species  $y$  depends on the other species biomass  $z$  (and vice versa) captures ecosystemic features of species interactions. Furthermore, these interactions are complicated by uncertainties  $\varepsilon_y$  and  $\varepsilon_z$ . After two periods,  $\varepsilon_y(t)$  indirectly impacts  $z(t+2)$  through  $y(t+1)$ , so that both disturbances affect both species. According to the nature of the interaction between  $y$  and  $z$ , uncertainties affecting one of the species will constitute lagged positive or negative externalities for the other species. Catches are given by  $v_y y \mathcal{R}_y(y, z, \varepsilon_y)$  and  $v_z z \mathcal{R}_z(y, z, \varepsilon_z)$  (measured in biomass). This model is generic in that no explicit or analytic assumptions are made on how the growth factors  $\mathcal{R}_y$  and  $\mathcal{R}_z$  indeed depend upon both biomasses  $(y, z)$  and upon the uncertainties  $(\varepsilon_y, \varepsilon_z)$ , except continuity.

Uncertainties  $(\varepsilon_y(t), \varepsilon_z(t))$  in (1) are assumed to take their values in a known two-dimensional set:

$$(\varepsilon_y(t), \varepsilon_z(t)) \in \mathbb{S}(t) \subset \mathbb{R}^2. \quad (2)$$

An uncertainty *scenario* is defined as a sequence of length  $T - t_0$  of uncertainty couples:

$$(\varepsilon_y(\cdot), \varepsilon_z(\cdot)) = ((\varepsilon_y(t_0), \varepsilon_z(t_0)), \dots, (\varepsilon_y(T-1), \varepsilon_z(T-1))) \in \prod_{t=t_0}^{T-1} \mathbb{S}(t). \quad (3)$$

Now, we propose to define *sustainability* as the ability to respect preservation and production minimal levels for all times, building upon the original approach of [Béné, Doyen, and Gabay, 2001]. For this purpose, we consider:

- on the one hand, *minimal biomass levels*  $y^b \geq 0$ ,  $z^b \geq 0$ , one for each species,

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<sup>1</sup>This approach can be easily extended to more than two species in interaction

- on the other hand, *minimal catch levels*  $Y^b \geq 0$ ,  $Z^b \geq 0$ , one for each species.

These figures are inputs to the robust viability kernel defined now.

Because it is backed on safety thresholds, the viability approach is particularly suited to the management of fisheries, which is increasingly governed by biological reference points constituting bottom line for stock depletion [Smith, Hunt, and Rivard, 1993]. Economic thresholds are assumed to be provided by policymakers rather than derived from a fishery production structure and demand model. However, it is possible to introduce such modelling component in the viability theoretical framework.

## 2.2 The robust viability kernel

To lay out the definition of the robust viability kernel, we need the notion of strategy. A control *strategy*  $\gamma$  is defined as a sequence of mappings from biomasses towards efforts as follows:

$$\gamma = \{\gamma_t\}_{t=t_0, \dots, T-1}, \quad \text{with} \quad \gamma_t : \mathbb{R}^2 \rightarrow [0, 1]^2. \quad (4)$$

A control strategy  $\gamma$  as in (4) and the dynamic model (1) jointly produce state paths by the initial state  $(y(t_0), z(t_0)) = (y_0, z_0)$  and the closed-loop dynamics

$$\begin{cases} y(t+1) &= y(t) \mathcal{R}_y(y(t), z(t), \varepsilon_y(t)) (1 - \gamma_t(y(t), z(t))) , \\ z(t+1) &= z(t) \mathcal{R}_z(y(t), z(t), \varepsilon_z(t)) (1 - \gamma_t(y(t), z(t))) , \end{cases} \quad (5)$$

and control paths by

$$(v_y(t), v_z(t)) = \gamma_t(y(t), z(t)) , \quad t = t_0, \dots, T-1. \quad (6)$$

Notice that, as in (6), controls  $(v_y(t), v_z(t))$  are determined by constantly adapting to the state  $(y(t), z(t))$  of the system, itself affected by past uncertainties and controls.

The *robust viability kernel*  $\text{Viab}^R(t_0)$  [De Lara and Doyen, 2008] is the set of initial states  $(y(t_0), z(t_0))$  for which there exists a control strategy  $\gamma$  as in (4), such that, for any uncertainty scenario  $(\varepsilon_y(\cdot), \varepsilon_z(\cdot)) \in \prod_{t=t_0}^{T-1} \mathbb{S}(t)$  in (3), the state path  $\{(y(t), z(t))\}_{t=t_0, \dots, T}$  as in (5), and control path  $\{(v_y(t), v_z(t))\}_{t=t_0, \dots, T-1}$  as in (6), satisfy the following goals:

- preservation (minimal biomass levels),  $\forall t = t_0, \dots, T$ ,

$$y(t) \geq y^b, \quad z(t) \geq z^b, \quad (7)$$

- production requirements (minimal catch levels),  $\forall t = t_0, \dots, T-1$ ,

$$v_y(t) y(t) \mathcal{R}_y(y(t), z(t), \varepsilon_y(t)) \geq Y^b, \quad v_z(t) z(t) \mathcal{R}_z(y(t), z(t), \varepsilon_z(t)) \geq Z^b. \quad (8)$$

States belonging to the robust viability kernel are also named *robust viable states*. Characterizing robust viable states makes it possible to test whether or not minimal biomass and catch levels can be guaranteed for all time, despite of uncertainty. By *guaranteed* we mean that biomasses and catches never fall below the minimal thresholds as in the inequalities (7) and (8).

The robust viability kernel can be computed numerically by means of a dynamic programming equation associated with dynamics (1), state constraints (7) and control constraints (8) (see §B in Appendix and [De Lara and Doyen, 2008]).

### 2.3 The deterministic viability kernel

The deterministic version of the framework exposed in §2.2 corresponds to the case where the uncertainties  $(\varepsilon_y(t), \varepsilon_z(t)) = (0, 0)$  for all  $t = t_0, \dots, T - 1$ , that is, the uncertainty sets in (2) are reduced to the singleton  $\mathbb{S}(t) = \{(0, 0)\}$ . In that case, the robust viability kernel coincides with the so-called *viability kernel*  $\text{Viab}(t_0)$  [Aubin, 1991], defined in §A in Appendix.

The following Proposition 1 gives an analytical expression of the deterministic viability kernel under conditions on the guaranteed levels in (7) and (8). The proof, adapted from [De Lara, Ocaña Anaya, and Ricardo Oliveros-Ramos, 2012], is given in §A in Appendix.

**Proposition 1** *If the minimal biomass thresholds  $y^b, z^b$  and catch thresholds  $Y^b, Z^b$  are such that*

$$y^b \mathcal{R}_y(y^b, z^b, 0) - y^b \geq Y^b \text{ and } z^b \mathcal{R}_z(y^b, z^b, 0) - z^b \geq Z^b, \quad (9)$$

*the deterministic viability kernel is given by*

$$\text{Viab}(t_0) = \left\{ (y, z) \in \mathbb{R}_+^2 \mid y \geq y^b, z \geq z^b, y \mathcal{R}_y(y, z, 0) - y \geq Y^b, z \mathcal{R}_z(y, z, 0) - z \geq Z^b \right\}. \quad (10)$$

The interpretation of conditions (9) is as follows. At the point  $(y^b, z^b)$  of minimum biomass thresholds, the surplus  $y^b \mathcal{R}_y(y^b, z^b, 0) - y^b \geq Y^b$  and  $z^b \mathcal{R}_z(y^b, z^b, 0) - z^b \geq Z^b$  are at least equal to the minimum catch thresholds  $Y^b$  and  $Z^b$ , respectively. Notice that the expression (10) does not depend on the horizon  $T$  (where  $T \geq t_0 + 2$ ): for any initial state in the deterministic viability kernel  $\text{Viab}(t_0)$ , there exists a strategy such that the constraints (7) and (8) are satisfied for all times from  $t_0$  to infinity.

## 3 Application to the Anchovy-Hake Couple in the Peruvian Upwelling Ecosystem (1971–1981)

Now, we apply a robust viability analysis to the Peruvian hake-anchovy fisheries between 1971 and 1981. For this, we extend the model in [De Lara, Ocaña Anaya, and Ricardo Oliveros-Ramos, 2012] to the uncertain



case. We compute the robust viability kernel numerically, testing different assumptions on the uncertainty sets  $\mathbb{S}(t)$  in (2), to appraise the sensitivity of the size and content of the robust viability kernel with respect to the set of uncertainty scenarios.

### 3.1 Lotka-Volterra dynamical model with uncertainties

The Peruvian anchovy-hake system is modeled as a prey-predator system, where the anchovy growth rate is decreasing in the hake population. We describe this interaction by the following discrete-time Lotka-Volterra dynamical system

$$\begin{aligned} y(t+1) &= \overbrace{y(t) \left( \varepsilon_y(t) + R - \frac{R}{\kappa} y(t) - \alpha z(t) \right)}^{\mathcal{R}_y(y(t), z(t), \varepsilon_y(t))} (1 - v_y(t)) \\ z(t+1) &= \underbrace{z(t) \left( \varepsilon_z(t) + L + \beta y(t) \right)}_{\mathcal{R}_z(y(t), z(t), \varepsilon_z(t))} (1 - v_z(t)), \end{aligned} \quad (11)$$

where  $R > 1$ ,  $0 < L < 1$ ,  $\alpha > 0$ ,  $\beta > 0$  and  $\kappa = \frac{R}{R-1}K$ , with  $K > 0$  the carrying capacity for the prey. The variable  $y$  stands for anchovy biomass and  $z$  for hake biomass. The model (11) is a *decision model* the purpose of which is not to provide detailed biological “knowledge” about the Peruvian upwelling ecosystem, but rather to capture the essential features of the system in what concerns decision making.

The five parameters of the deterministic version of the Lotka-Volterra model (that is, with  $\varepsilon_y(t) = 0$  and  $\varepsilon_z(t) = 0$  in the dynamical system (11)) have been estimated in [De Lara, Ocaña Anaya, and Ricardo Oliveros-Ramos, 2012], based on 11 yearly observations of the Peruvian anchovy-hake biomasses and catches over the time period 1971–1981. Their values are given in Table 1.

Parameters	Estimates
$R$	$2.25 \text{ year}^{-1}$
$L$	$0.945 \text{ year}^{-1}$
$\kappa$	$67113 \text{ } 10^3 \text{ tons}$
$K$	$37285 \text{ } 10^3 \text{ tons}$
$\alpha$	$1.220 \text{ } 10^{-6} \text{ tons}^{-1}$
$\beta$	$4.845 \text{ } 10^{-8} \text{ tons}^{-1}$

Table 1: Parameters of the Lotka-Volterra model (11)

Following [IMARPE, 2000, 2004], we consider the minimal biomasses  $y^b = 7,000,000$  tons and  $z^b = 200,000$  tons in (7), and minimal catches  $Y^b = 2,000,000$  tons and  $Z^b = 5,000$  tons in (8). The condition (9) in Proposition 1 is satisfied for the above minimal threshold values and for the Lotka-Volterra model parameters in Table 1. Therefore, we can exactly compute the deterministic viability kernel.

## 3.2 Choice of uncertainty sets

Now, we specify the uncertainty sets  $\mathbb{S}(t)$  in (2), in which the uncertainties  $\varepsilon_y(t)$  and  $\varepsilon_z(t)$  in (11) take their values. For the sake of simplicity, we consider stationary uncertainty sets  $\mathbb{S} = \mathbb{S}(t)$ , though this feature is not required for a dynamic programming equation to hold true.

First, we form an uncertainty set  $\mathbb{S}_E$  with empirical values. Second, we refine this set. Third, we identify and only consider extreme uncertainties producing worst-case scenarios. In §3.3, we will explain these choices in light of the corresponding robust viability kernels.

### 3.2.1 Empirical uncertainties set and a refinement

Figure 1 depicts the observed biomasses of Peruvian anchovy and hake over the years 1971–1981 and the simulated biomasses with the deterministic version of the Lotka-Volterra model (that is, with  $\varepsilon_y(t) = 0$  and  $\varepsilon_z(t) = 0$  in the dynamical system (11)) and given the observed harvesting efforts over years 1971–1981<sup>2</sup>. The time period 1971–1981 is denoted by  $t = t_0, \dots, T$ , with  $t_0 = 0$ , and  $T = 10$ . Let  $(\bar{y}(t), \bar{z}(t))_{t=t_0, \dots, T}$  and

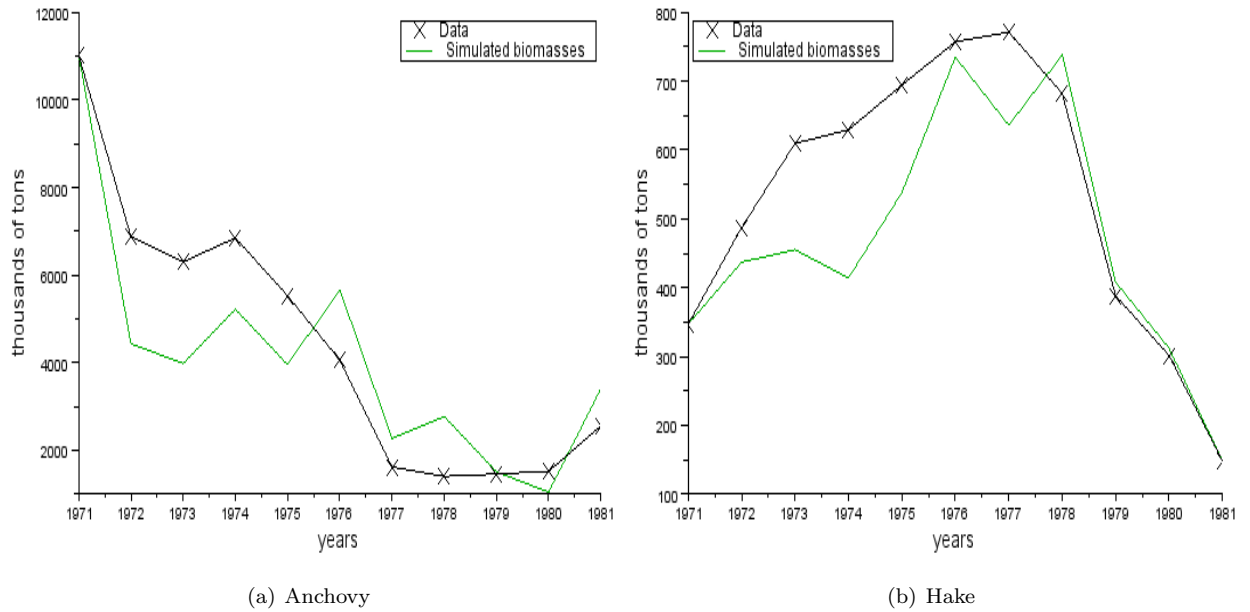


Figure 1: Observed and simulated biomasses over 1971–1981

$(\bar{y}_y(t), \bar{v}_z(t))_{t=t_0, \dots, T-1}$  denote the observed biomass and effort trajectories. We set  $\bar{\varepsilon}_y(t)$  and  $\bar{\varepsilon}_z(t)$  implicitly

<sup>2</sup>Precisely, the biomass couple estimated in 1971 constitutes our starting state for simulating species biomasses. We plug this initial estimate of the anchovy–hake biomass couple and the 1971 catch values of each species in the deterministic version of the Lotka-Volterra model described in (11). This allows us to simulate the value of both biomasses in the following period. We renew this operation for each date until 1981, except that the current biomass couple we plug in the model the simulated one, while we apply the estimated catch couple of the current date all along.

defined by

$$\begin{cases} \bar{y}(t+1) &= \bar{y}(t)(\bar{\varepsilon}_y(t) + R - \frac{R}{\kappa}\bar{y}(t) - \alpha\bar{z}(t))(1 - \bar{v}_y(t)) \\ \bar{z}(t+1) &= \bar{z}(t)(\bar{\varepsilon}_z(t) + L + \beta\bar{y}(t))(1 - \bar{v}_z(t)), \end{cases} \quad (12)$$

so that (11) is satisfied. Figure 2 displays the points  $\{(\bar{\varepsilon}_y(t), \bar{\varepsilon}_z(t)) | t = t_0, \dots, T-1\}$ , (there are 10 points as 1971 observations are used as starting points for simulating biomasses). We denote  $\bar{\varepsilon}_y^{min} = \min_t \bar{\varepsilon}_y(t) = -0.25$ ,  $\bar{\varepsilon}_y^{max} = \max_t \bar{\varepsilon}_y(t) = 1.54$ ,  $\bar{\varepsilon}_z^{min} = \min_t \bar{\varepsilon}_z(t) = -0.38$  and  $\bar{\varepsilon}_z^{max} = \max_t \bar{\varepsilon}_z(t) = 0.088$ .

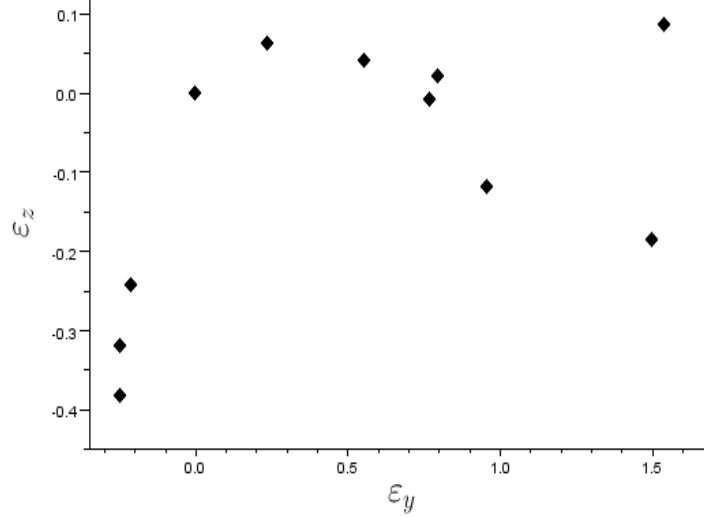


Figure 2: Empirical uncertainties  $(\bar{\varepsilon}_y(t), \bar{\varepsilon}_z(t))_{t=t_0, \dots, T-1}$  characterized by (12)

- The *empirical uncertainties set*

$$\mathbb{S}_E = \{(\bar{\varepsilon}_y(t), \bar{\varepsilon}_z(t)) | t = t_0, \dots, T-1\} \cup \{(0, 0)\} \quad (13)$$

is made of the ten empirical uncertainty couples (see diamonds in Figure 2) and the uncertainty couple  $(\varepsilon_y, \varepsilon_z) = (0, 0)$  (corresponding to the deterministic case).

- The *refined empirical uncertainties set*  $\mathbb{S}_{ER}$  is made of 900 uncertainty couples produced by a  $30 \times 30$  grid over the surface  $[\bar{\varepsilon}_y^{min}, \bar{\varepsilon}_y^{max}] \times [\bar{\varepsilon}_z^{min}, \bar{\varepsilon}_z^{max}]$ , including all the uncertainty couples of  $\mathbb{S}_E$  (see the grid in Figure 3).

### 3.2.2 Uncertainty sets reduced to extreme values

Through numerical simulations, we found that the set of robust viable states is sensitive to few extreme points of the uncertainty set  $\mathbb{S}_{ER}$ . This is why, in addition to  $\mathbb{S}_E$  and  $\mathbb{S}_{ER}$ , we consider the following two uncertainty sets,  $\mathbb{S}_M$  and  $\mathbb{S}_H$ .

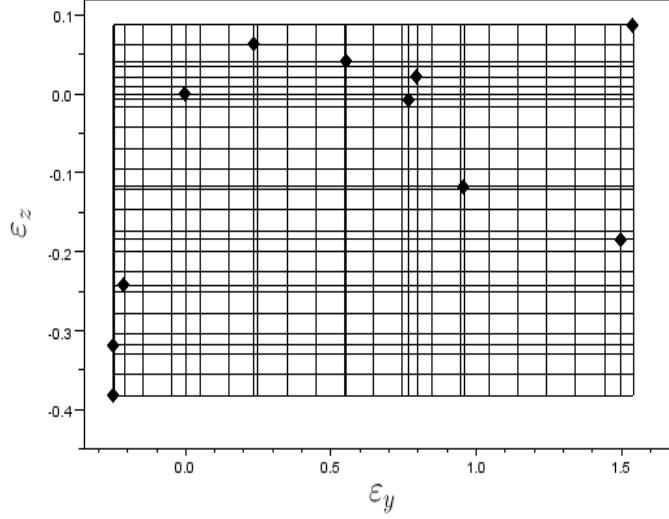


Figure 3: Uncertainty sets  $\mathbb{S}_E$  (diamonds) and  $\mathbb{S}_{ER}$  (grid)

- The uncertainty set  $\mathbb{S}_M$  is composed of two extreme uncertainty couples taken from the set  $\mathbb{S}_{ER}$ :

$$\mathbb{S}_M = \{(\bar{\varepsilon}_y^{min}, \bar{\varepsilon}_z^{min}), (\bar{\varepsilon}_y^{min}, \bar{\varepsilon}_z^{max})\} \subset \mathbb{S}_{ER} . \quad (14)$$

- The uncertainty set  $\mathbb{S}_H$  is obtained by increasing the values in  $\mathbb{S}_M$  by 20%:

$$\mathbb{S}_H = 1.2 * \mathbb{S}_M . \quad (15)$$

The uncertainty couple  $(\bar{\varepsilon}_y^{min}, \bar{\varepsilon}_z^{min})$  corresponds to low growth factor for both species, whereas  $(\bar{\varepsilon}_y^{min}, \bar{\varepsilon}_z^{max})$  affects negatively the prey growth and positively the predator growth.

### 3.3 Discussion on the viability kernels

We introduced a dynamical model of harvested ecosystem in the Peruvian upwelling and sustainability constraints in §3.1. In §3.2, we laid out different sets of uncertainties affecting this dynamics. These ingredients will allow us to compute robust viability kernels for various uncertainty sets (including the deterministic case). In §3.3.1, we compare the viability kernels: the deterministic, the robust resulting from the uncertainty set  $\mathbb{S}_E$  and that obtained from the uncertainty set  $\mathbb{S}_{ER}$ . In §3.3.2, we turn to the uncertainty sets  $\mathbb{S}_M$  and  $\mathbb{S}_H$ , built upon “extreme uncertainties” and we scrutinize how these sets impact the robust viability kernels.

### 3.3.1 Robust viability kernel and empirical uncertainties

Replacing the growth rates  $\mathcal{R}_y$  and  $\mathcal{R}_z$  in (10) by their expressions (11) yields the expression of the deterministic viability kernel:

$$\begin{aligned} \text{Viab}(t_0) &= \left\{ (y, z) \mid y \geq y^b, z \geq z^b, y\left(R - \frac{R}{\kappa}y - \alpha z\right) - y^b \geq Y^b, z(L + \beta y) - z^b \geq Z^b \right\} \\ &= \left\{ (y, z) \mid y \geq y^b, \frac{1}{\alpha}\left[R - \frac{R}{\kappa}y - \frac{y^b + Y^b}{y}\right] \geq z \geq \max\left\{\frac{z^b + Z^b}{L + \beta y}, z^b\right\} \right\}. \end{aligned} \quad (16)$$

In §B in Appendix, we detail how the robust viability kernels are computed numerically, with the scientific software Scicoslab. Figure 4 displays the deterministic viability kernel and the robust viability kernels associated with dynamics (11), constraints (7) and (8), and with the uncertainty sets  $\mathbb{S}_E$  and  $\mathbb{S}_{ER}$ , respectively. The horizontal and vertical lines represent the minimal biomass safety levels  $y^b$  and  $z^b$ .

The humped shape of the upper frontier of the deterministic viability kernel in Figure 4 stems from the logistic dynamics of the anchovy stock. Indeed, from the expression of  $\text{Viab}(t_0)$  in (16), we deduce that the upper frontier is characterized by

$$\frac{1}{\alpha}\left[R - \frac{R}{\kappa}y - \frac{y^b + Y^b}{y}\right] = z \Leftrightarrow y\left(R - \frac{R}{\kappa}y - \alpha z\right) = Y^b + y^b \Leftrightarrow y\mathcal{R}_y(y, z, 0) = Y^b + y^b.$$

Hence, before a tipping anchovy biomass level  $\bar{y}(z) = \frac{\kappa(R - \alpha z)}{2R}$ , the future biomass  $y\mathcal{R}_y(y, z, 0)$  increases with  $y$ , whereas it decreases after.

#### Gap between the deterministic kernel and the robust ones

In Figure 4, we observe an important gap between the deterministic kernel and the robust ones. A share of the states identified as viable by the deterministic approach — those below the upper curve and above the dotted lines in Figure 4 — is in fact excluded when uncertainty is taken into account. Indeed, there exists no effort strategy that can guarantee preservation and production minima for biomass couples standing outside the robust kernels, given the chosen scenarios sets and time horizon. Furthermore, we cannot tell whether the effort strategies advocated by the deterministic approach for an initial biomass couple belonging to the robust kernels guarantee sustainability objectives over time in presence of uncertainty.

#### Sensitivity of the robust viability kernel to uncertainty sets

Since  $\{(0, 0)\} \subset \mathbb{S}_E \subset \mathbb{S}_{ER}$ , where the uncertainty sets  $\mathbb{S}_E$  and  $\mathbb{S}_{ER}$  are given in §3.2.1, we expect the corresponding robust and deterministic viability kernels to satisfy

$$\text{Viab}_{ER}^R(t_0) \subset \text{Viab}_E^R(t_0) \subset \text{Viab}(t_0). \quad (17)$$

We indeed observe strict inclusions in Figure 4. This confirms our initial guess that, by exposing the ecosystem dynamics to a denser set of scenarios  $\mathbb{S}_{ER}$  instead of  $\mathbb{S}_E$ , fewer initial states should be likely to allow for an effort strategy guaranteeing all sustainability constraints at all times.

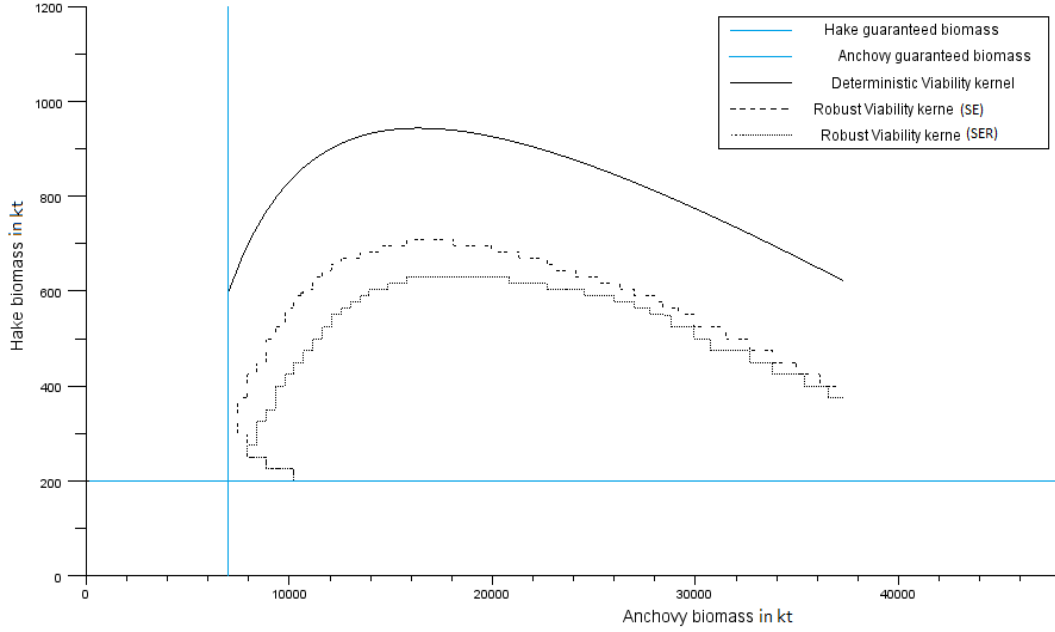


Figure 4: Deterministic and robust viability kernels associated with the uncertainty sets  $\mathbb{S}_E$  and  $\mathbb{S}_{ER}$

In addition, we examine the sensitivity of the robust viability kernel  $\text{Viab}_{ER}^R(t_0)$ , to the length of the time horizon. It appears that beyond 7 years  $T \geq 7$ , the set of robust viable states is stable.

### 3.3.2 Robust viability kernel and extreme uncertainties

Figure 5 displays the deterministic viability kernel (16) once again, and the two robust viability kernels associated with dynamics (11), constraints (7) and (8), and with the uncertainty sets  $\mathbb{S}_M$  and  $\mathbb{S}_H$ , respectively, as defined in §3.2.2.

#### Extreme uncertainties

Since  $\mathbb{S}_M \subset \mathbb{S}_{ER}$ , we know that:

$$\text{Viab}_{ER}^R(t_0) \subset \text{Viab}_M^R(t_0). \quad (18)$$

However, in practice the inclusion is not strict: our numerical results show that the robust viability kernels  $\text{Viab}_M^R(t_0)$  and  $\text{Viab}_E^{ER}(t_0)$  are equal. More precisely, whatever the set of uncertainty couples we add to  $\mathbb{S}_M$ , with values in the rectangle  $[\bar{\varepsilon}_y^{\min}, \bar{\varepsilon}_y^{\max}] \times [\bar{\varepsilon}_z^{\min}, \bar{\varepsilon}_z^{\max}]$ , the resulting robust viability kernels are the same. On the other hand, when we eliminated one of the two uncertainty couples in  $\mathbb{S}_M$ , we observed that the robust viability kernel increased.

The fact that the couple  $(\bar{\varepsilon}_y^{min}, \bar{\varepsilon}_z^{max})$  produces worse adverse ecological and economic consequences is quite intuitive, whereas it is less obvious for the couple  $(\bar{\varepsilon}_y^{min}, \bar{\varepsilon}_z^{min})$ , given the nonlinear relationships linking both species. Indeed, prey-predator interaction introduces a trade-off between fish stocks levels in the sense that the enhancement of a biomass necessarily takes place at the expense of the other. Thereby, a positive shock to the biomass of the predator species does not produce an ecological improvement of the ecosystem, especially if the biomass of the prey is undermined alongside. On the other hand, if the relative abundance of both stocks is affected in the same direction, it is less clear why the ecosystem reaches its most critical state given the antagonist relation linking both fish stock.

### Expended extreme uncertainties

Now, we consider the uncertainty set  $\mathbb{S}_M$  and the corresponding viability kernel  $\text{Viab}_M^R(t_0)$ . By numerical simulations, we explore the sensitivity of  $\text{Viab}_M^R(t_0)$  to changes in extreme uncertainties values.

- When, we increase  $\bar{\varepsilon}_z^{max}$ , all other things kept equal in  $\mathbb{S}_M$ , we observe that the viability kernel is enlarged.
- When, we increase (in absolute value)  $\bar{\varepsilon}_y^{min}$  and  $\bar{\varepsilon}_z^{min}$  simultaneously, all other things kept equal in  $\mathbb{S}_M$ , the viability kernel is empty beyond a 25% increase of these two extreme uncertainties.
- When we increase all uncertainties in  $\mathbb{S}_M$  by more than 20% ( a 20% increase corresponds to  $\mathbb{S}_H$ ), the robust viability kernel is empty.

Thus, the viability kernel displays contrasted patterns when submitted to different increases in extreme uncertainty values. A possible explanation comes from (3), which reflects an "independence" assumption of uncertainties w.r.t time. Due to this assumption, scenarios can display arbitrary evolutions, switching from one extreme to another between time periods. Such scenarios deserve the label of worst-case scenarios as they narrow the possibility of guaranteeing ecological and economic objectives at all times. Hence, amplifying the distance between our extreme uncertainties shrinks the robust viability kernel.

### Retrospective analysis of the Peruvian Anchovy-Hake fisheries trajectories between 1971 and 1981

In Figure 5, the circles indicate the biomass observations of the anchovy-hake couple over 1971–1981. Only one circle, marked by a cross, stands within the robust viability kernel  $\text{Viab}_M^R(t_0)$ , corresponding to the initial biomass couple observed in 1971. Starting from that date, there theoretically existed a harvest strategy providing, for the next 10 years, at least the sustainable yields  $Y^b = 2,000,000$  tons and  $Z^b = 5,000$  tons, and guaranteeing biomasses over the preservation thresholds  $y^b = 7,000,000$  tons,  $z^b = 2,000,000$  tons, whatever the uncertainties stemming from  $\mathbb{S}_H$ , or more exactly from the rectangle  $[\bar{\varepsilon}_y^{min}, \bar{\varepsilon}_y^{max}] \times [\bar{\varepsilon}_z^{min}, \bar{\varepsilon}_z^{max}]$ .

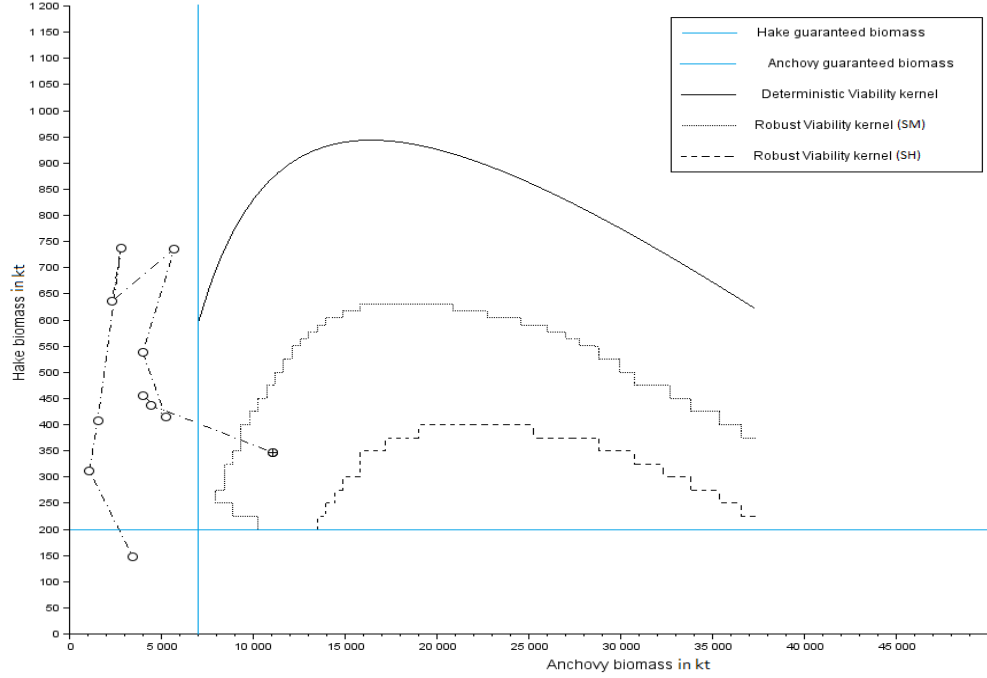


Figure 5: Comparing the deterministic and robust viable kernels associated with uncertainty sets  $\mathbb{S}_M$  and  $\mathbb{S}_H$

In reality, the catches of year 1971 were very high, and the biomass trajectories were well below the biological minimal levels for 14 years.

## 4 Conclusion

This work is a theoretical and practical contribution to ecosystem sustainable management under uncertainty. The robust viable kernel is an insightful mean to display the impact of uncertainty on the possibility of a sustainable management. Wherever a fishery stands, the set of robust states enables to foretell whether economic and ecological objectives can be guaranteed over a time span, despite of uncertainty.

For the anchovy-hake couple in the Peruvian upwelling ecosystem, we have shown to what extent taking into account uncertainty affects the conclusions drawn from the deterministic case. By making allowance for uncertainties in the ecosystem dynamics, effort strategies guaranteeing all sustainability constraints at all times exist for fewer initial states than in the deterministic case.

In addition, we have been able to shed light on the uncertainties that really matter for a precautionary approach. Indeed, by computing several robust viable kernels, we have realized that only few important uncertainties matter, and that they correspond to extreme cases. What is more, we have shown that not



only the absolute value of extreme uncertainties matters, but also the distance between them. Assessing which uncertainties truly impact the robust viability kernel can help the decision-maker to focus on those uncertainties that are relevant for sustainable management.

In rather common situations where very little is known about uncertainties, the robust framework contents itself of poor assumptions on sets rather than possibly unjustified probabilistic ones. However, we have seen that the robust viability kernel can be empty. To account for less radical analysis, the viability stochastic theory is an alternative approach to address dynamical control problems under uncertainty and constraints. This approach allows for constraints violations with a low probability. This issue is under current investigation.

## A The Deterministic Viability Kernel

The *deterministic viability kernel*,  $\mathbb{V}_{\text{iab}}(t_0)$ , associated with the following dynamics (19), and constraints (20) and (21), for  $t = t_0, \dots, T$ , is the set of viable states defined as follows. A couple  $(y_0, z_0)$  of initial biomasses is said to be a *viable state* if there exist a trajectory of harvesting efforts (controls)  $(v_y(t), v_z(t)) \in [0, 1]$ ,  $t = t_0, \dots, T-1$ , such that the state path  $\{(y(t), z(t))\}_{t=t_0, \dots, T}$ , and control path  $\{(v_y(t), v_z(t))\}_{t=t_0, \dots, T-1}$ , solution of<sup>3</sup>

$$\begin{cases} y(t+1) &= y(t)\mathcal{R}_y(y(t), z(t))(1 - v_y(t)) , \\ z(t+1) &= z(t)\mathcal{R}_z(y(t), z(t))(1 - v_z(t)) , \end{cases} \quad (19)$$

starting from  $(y(t_0), z(t_0)) = (y_0, z_0)$  satisfy the following goals:

- preservation (minimal biomass levels): for all  $t = t_0, \dots, T$

$$y(t) \geq y^b, \quad z(t) \geq z^b, \quad (20)$$

- and production requirements (minimal catch levels): for all  $t = t_0, \dots, T-1$

$$v_y(t)y(t)\mathcal{R}_y(y(t), z(t)) \geq Y^b, \quad v_z(t)z(t)\mathcal{R}_z(y(t), z(t)) \geq Z^b, \quad (21)$$

We now turn to the proof of Proposition 1 in §2.3.

**Proof.** Consider  $y^b \geq 0$ ,  $z^b \geq 0$ ,  $Y^b \geq 0$ ,  $Z^b \geq 0$ . We set

$$\mathbb{V}_0 = \left\{ (y, z) \in \mathbb{R}_+^2 \mid y \geq y^b, z \geq z^b \right\}$$

and we define a sequence  $(\mathbb{V}_k)_{k \in \mathbb{N}}$  inductively by

$$\begin{aligned} \mathbb{V}_{k+1} &= \{ (y, z) \in \mathbb{V}_k \mid \exists (v_y, v_z) \in [0, 1] \text{ such that } yv_y\mathcal{R}_y(y, z) \geq Y^b, zv_z\mathcal{R}_z(y, z) \geq Z^b, \\ &\quad \text{and } y' = y\mathcal{R}_y(y, z)(1 - v_y), z' = z\mathcal{R}_z(y, z)(1 - v_z), \\ &\quad \text{are such that } (y', z') \in \mathbb{V}_k \} . \end{aligned}$$

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<sup>3</sup>Equation (19) is (1) with the uncertainty couple  $(\varepsilon_y, \varepsilon_z) = (0, 0)$  (corresponding to the deterministic case). Notice that the growth rates  $\mathcal{R}_y$  and  $\mathcal{R}_z$  do not include uncertainty variables, as was the case in §2.1.

For  $k = 0$ , we obtain

$$\begin{aligned}
\mathbb{V}_1 &= \left\{ (y, z) \left| \begin{array}{l} y \geq y^b, z \geq z^b \text{ and, for some } (v_y, v_z) \in [0, 1], \\ v_y y \mathcal{R}_y(y, z) \geq Y^b, v_z z \mathcal{R}_z(y, z) \geq Z^b, \\ y \mathcal{R}_y(y, z)(1 - v_y) \geq y^b, z \mathcal{R}_z(y, z)(1 - v_z) \geq z^b \end{array} \right. \right\} \\
&= \left\{ (y, z) \left| \begin{array}{l} y \geq y^b, z \geq z^b \text{ for which there exist } (v_y, v_z) \text{ such that} \\ \frac{Y^b}{y \mathcal{R}_y(y, z)} \leq v_y \leq \frac{y \mathcal{R}_y(y, z) - y^b}{y \mathcal{R}_y(y, z)} \quad \text{and} \quad 0 \leq v_y \leq 1, \\ \frac{Z^b}{z \mathcal{R}_z(y, z)} \leq v_z \leq \frac{z \mathcal{R}_z(y, z) - z^b}{z \mathcal{R}_z(y, z)} \quad \text{and} \quad 0 \leq v_z \leq 1 \end{array} \right. \right\} \\
&= \left\{ (y, z) \left| \begin{array}{l} y \geq y^b, z \geq z^b, \\ \sup\{0, \frac{Y^b}{y \mathcal{R}_y(y, z)}\} \leq \inf\{1, 1 - \frac{y^b}{y \mathcal{R}_y(y, z)}\} \\ \sup\{0, \frac{Z^b}{z \mathcal{R}_z(y, z)}\} \leq \inf\{1, 1 - \frac{z^b}{z \mathcal{R}_z(y, z)}\} \end{array} \right. \right\} \\
&= \left\{ (y, z) \left| y \geq y^b, z \geq z^b, \frac{Y^b}{y \mathcal{R}_y(y, z)} \leq \frac{y \mathcal{R}_y(y, z) - y^b}{y \mathcal{R}_y(y, z)}, \frac{Z^b}{z \mathcal{R}_z(y, z)} \leq \frac{z \mathcal{R}_z(y, z) - z^b}{z \mathcal{R}_z(y, z)} \right\} \\
&= \left\{ (y, z) \left| y \geq y^b, z \geq z^b, Y^b \leq y \mathcal{R}_y(y, z) - y^b, Z^b \leq z \mathcal{R}_z(y, z) - z^b \right\}.
\end{aligned}$$

Then, for  $k = 1$ , we obtain

$$\begin{aligned}
\mathbb{V}_2 &= \left\{ (y, z) \left| \begin{array}{l} y \geq y^b, z \geq z^b \text{ and, for some } (v_y, v_z) \in [0, 1], \\ v_y y \mathcal{R}_y(y, z) \geq Y^b, v_z z \mathcal{R}_z(y, z) \geq Z^b \\ \text{and such that } (y', z') \in \mathbb{V}_1 \\ \text{where } y' = y \mathcal{R}_y(y, z)(1 - v_y), z' = z \mathcal{R}_z(y, z)(1 - v_z) \end{array} \right. \right\} \\
&= \left\{ (y, z) \left| \begin{array}{l} y \geq y^b, z \geq z^b \text{ and, for some } (v_y, v_z) \in [0, 1], \\ v_y y \mathcal{R}_y(y, z) \geq Y^b, v_z z \mathcal{R}_z(y, z) \geq Z^b, y' \geq y^b, z' \geq z^b, \\ Y^b \leq y' \mathcal{R}_y(y', z') - y^b, Z^b \leq z' \mathcal{R}_z(y', z') - z^b \\ \text{where } y' = y \mathcal{R}_y(y, z)(1 - v_y), z' = z \mathcal{R}_z(y, z)(1 - v_z) \end{array} \right. \right\}.
\end{aligned}$$

We now make use of the property (see [De Lara, Ocaña Anaya, and Ricardo Oliveros-Ramos, 2012]) that, when the decreasing sequence  $(\mathbb{V}_k)_{k \in \mathbb{N}}$  is stationary, its limit is the viability kernel  $\mathbb{V}_{\text{iab}}(t_0)$ . Hence, it suffices to show that  $\mathbb{V}_1 \subset \mathbb{V}_2$  to obtain that  $\mathbb{V}_{\text{iab}}(t_0) = \mathbb{V}_1$ .

Let  $(y, z) \in \mathbb{V}_1$ . We have that

$$y \geq y^b, \quad z \geq z^b \text{ and } y \mathcal{R}_y(y, z) - y^b \geq Y^b, \quad z \mathcal{R}_z(y, z) - z^b \geq Z^b.$$

Let us set  $\hat{v}_y = \frac{y \mathcal{R}_y(y, z) - y^b}{y \mathcal{R}_y(y, z)}$ , which has the property that  $y' = y \mathcal{R}_y(y, z)(1 - \hat{v}_y) = y^b$ . We prove that  $\hat{v}_y \in [0, 1]$ . Indeed, on the one hand, we have that  $\hat{v}_y \leq 1$  since  $1 - \hat{v}_y = y^b / y \mathcal{R}_y(y, z)$ , where  $y^b \geq 0$ . On the other hand, since by assumption  $y \mathcal{R}_y(y, z) - y^b \geq Y^b \geq 0$ , we deduce that  $\hat{v}_y \geq 0$ . The same holds true for  $\hat{v}_z$  and  $z' = z \mathcal{R}_z(y, z)(1 - \hat{v}_z) = z^b$ . By (9), we deduce that

$$y' \mathcal{R}_y(y', z') - y^b = y^b \mathcal{R}_y(y^b, z^b) - y^b \geq Y^b \text{ and } z' \mathcal{R}_z(y', z') - z^b = z^b \mathcal{R}_z(y^b, z^b) - z^b \geq Z^b.$$

The inclusion  $\mathbb{V}_1 \subset \mathbb{V}_2$  follows, hence  $\mathbb{V}_{iab}(t_0) = \mathbb{V}_1$ , and (10) holds true.  $\square$

The *viable controls* attached to a given viable state  $(y, z) \in \mathbb{V}_{iab}(t_0)$  are the admissible controls  $(v_y, v_z)$  such that the image by the dynamics (19) is in  $\mathbb{V}_{iab}(t_0)$ .

**Corollary 2** *Suppose that the assumptions of Proposition 1 are satisfied. The set of viable controls associated with the state  $(y, z)$  is*

$$\left\{ (v_y, v_z) \in [0, 1]^2 \left| \begin{array}{l} \frac{y\mathcal{R}_y(y, z) - y^b}{y\mathcal{R}_y(y, z)} \geq v_y \geq \frac{Y^b}{y\mathcal{R}_y(y, z)}, \quad \frac{z\mathcal{R}_z(y, z) - z^b}{z\mathcal{R}_z(y, z)} \geq v_z \geq \frac{Z^b}{z\mathcal{R}_z(y, z)}, \\ y'\mathcal{R}_y(y', z') - y^b \geq Y^b, \quad z'\mathcal{R}_z(y', z') - z^b \geq Z^b \end{array} \right. \right\},$$

where  $y' = y\mathcal{R}_y(y, z)(1 - v_y)$ ,  $z' = z\mathcal{R}_z(y, z)(1 - v_z)$ .

## B Numerical Computation of Robust Viability Kernels

We first sketch how to establish a dynamic programming equation associated with dynamics (1), and preservation (7) and production (8) minimal thresholds. Then, we depict a numerical discretization scheme to solve this equation numerically.

### B.1 Dynamic programming equation

The dynamic programming equation associated with dynamics (1), and preservation (7) and production (8) minimal thresholds is given by<sup>4</sup>

$$\begin{aligned} V_T(y, z) &= \mathbf{1}_{\mathbb{A}}(y, z), \\ V_t(y, z) &= \mathbf{1}_{\mathbb{A}}(y, z) \max_{(v_y, v_z) \in [0, 1]^2} \min_{(\varepsilon_y, \varepsilon_z) \in \mathbb{S}(t)} \left[ \mathbf{1}_{\mathbb{B}(y, z, \varepsilon_y, \varepsilon_z)}(v_y, v_z) V_{t+1}(G(y, z, v_y, v_z, \varepsilon_y, \varepsilon_z)) \right], \end{aligned} \quad (22)$$

for all  $t = t_0, \dots, T - 1$ , where the continuous function  $G$  denotes the dynamics (1)

$$G(y, z, v_y, v_z, \varepsilon_y, \varepsilon_z) = \begin{cases} y\mathcal{R}_y(y, z, \varepsilon_y)(1 - v_y), \\ z\mathcal{R}_z(y, z, \varepsilon_z)(1 - v_z), \end{cases} \quad (23)$$

where  $\mathbb{A}$  stands for the subset of biomass satisfying conservation objectives (7)

$$\mathbb{A} = \{(y, z) \mid y \geq y^b, z \geq z^b\} = [y^b, +\infty[ \times [z^b, +\infty[, \quad (24)$$

and where  $\mathbb{B}(y, z, \varepsilon_y, \varepsilon_z)$  stands for the subset of catches satisfying minimal production requirements (8)

$$\mathbb{B}(y, z, \varepsilon_y, \varepsilon_z) = \{(v_y, v_z) \in [0, 1]^2 \mid v_y y \mathcal{R}_y(y, z, \varepsilon_y) \geq Y^b, v_z z \mathcal{R}_z(y, z, \varepsilon_z) \geq Z^b\}. \quad (25)$$

The notation  $\mathbf{1}_{\mathbb{A}}(y, z)$  is the indicator function of the set  $\mathbb{A}$ : it takes the value 1 when  $(y, z) \in \mathbb{A}$  and 0 else. The same holds for  $\mathbf{1}_{\mathbb{B}(y, z, \varepsilon_y, \varepsilon_z)}(v_y, v_z)$ .

---

<sup>4</sup>What follows is a simple extension of the results in [De Lara and Doyen, 2008] and [Doyen and De Lara, 2010].

It turns out that, for all  $t = t_0, \dots, T$ , the robust viability value function  $V_t$  is the indicator function  $\mathbf{1}_{\text{Viab}^R(t)}$  of the robust viability kernel  $\text{Viab}^R(t)$  (see [De Lara and Doyen, 2008]). The sketch of the proof is as follows, by backward induction.

By (22), we have that  $V_T = \mathbf{1}_{\mathbb{A}} = \mathbf{1}_{\text{Viab}^R(T)}$ . Now, assume that  $V_{t+1} = \mathbf{1}_{\text{Viab}^R(t+1)}$ . When the operation  $\min_{(\varepsilon_y, \varepsilon_z) \in \mathbb{S}(t)}$  is performed in (22), the result is 1 if, and only if, for all uncertainties  $(\varepsilon_y, \varepsilon_z) \in \mathbb{S}(t)$ , we have both  $\mathbf{1}_{\mathbb{B}(y, z, \varepsilon_y, \varepsilon_z)}(v_y, v_z) = 1$  and  $\mathbf{1}_{\text{Viab}^R(t)}(G(y, z, v_y, v_z, \varepsilon_y, \varepsilon_z)) = 1$ , that is, both efforts  $(v_y, v_z)$  satisfy minimal production requirements (8) and the images  $G(y, z, v_y, v_z, \varepsilon_y, \varepsilon_z)$  by the dynamics  $G$  belong to the viability kernel  $\text{Viab}^R(t)$ . Then, the operation  $\max_{(v_y, v_z) \in [0,1]^2}$  yields 1 if, and only if, there is at least one control  $(v_y, v_z)$  — indeed achieved by continuity of the dynamics  $G$  in (23) — such that (8) is satisfied and  $G(y, z, v_y, v_z, \varepsilon_y, \varepsilon_z) \in \text{Viab}^R(t)$ . The term  $\mathbf{1}_{\mathbb{A}}(y, z) = 1$  if, and only if, the conservation objectives (7) are satisfied. To end, we obtain that  $V_t(y, z) = 1$  if, and only if, there exists at least one control  $(v_y, v_z)$  such that the conservation objectives (7) and the production requirements (8) are satisfied, and that the images  $G(y, z, v_y, v_z, \varepsilon_y, \varepsilon_z)$  by the dynamics  $G$  belong to the viability kernel  $\text{Viab}^R(t)$  for all uncertainties  $(\varepsilon_y, \varepsilon_z) \in \mathbb{S}(t)$ . By a simple extension of the results in [De Lara and Doyen, 2008] and [Doyen and De Lara, 2010], we have just characterized  $\text{Viab}^R(t)$ .

## B.2 Numerical resolution of the dynamic programming equation

Now, we expose how we proceed to find the robust viability kernel numerically thanks to the dynamic programming equation (22).

We discretize biomass, harvesting effort and uncertainty values. A top loop for time steps embraces two nested loops for state variables  $y$  and  $z$ , respectively. Next, loops over uncertainties nested in loops over harvesting efforts allow us to obtain the set of images associated with a biomass couple (some of these steps are actually done through matrix computing). Images for target constraints that are not satisfied are set equal to zero. We then project these images on the value function grid of the previous period, through linear interpolation. At given efforts, we retain the minimum value obtained over all uncertainty couples. Then, we retain the highest value produced by an effort couple among all tested. It is this value that is multiplied with the value function of the current time period, at the location of the biomass couple at stake. The robust viability kernel is defined as the set of grid points where the value function is equal to 1. This implies that biomass couples for which, at a date  $t$ , all images do not fall between four 1 in the interpolation are excluded from the robust viability kernel (in the sense that we provide robustness with respect to grid approximation).

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